

Lecture 6: Solar Cell Parameters and Equivalent Circuit

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Recap: Diode Equation

$$I = I_0 \left[\exp \left(\frac{qV}{k_B T} \right) - 1 \right] - I_{ph}$$

proportional to
photon flux

Thermal voltage:
 $V_{th} = k_B T / q = 26 \text{ mV at } 25^\circ\text{C}$

where

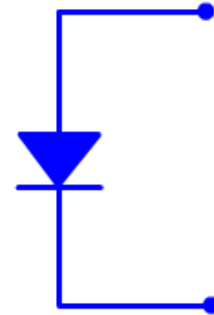
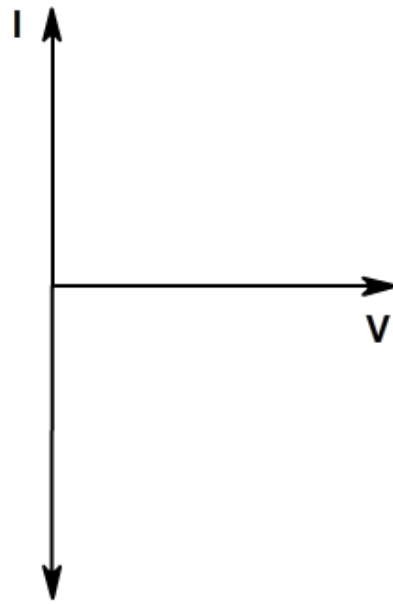
- I_{ph} = photocurrent (A)
- I_0 = diode saturation (dark) current (A)
- k_B = Boltzmann constant = $1.38 \times 10^{-23} \text{ J/K}$
- q = electronic charge = $1.6 \times 10^{-19} \text{ C}$

Typical numbers: $I_0 \cong 1 \text{ nA} - 1 \text{ mA}$, $I_{ph} \cong 3 - 10 \text{ A}$

Short Circuit: $V = 0$, $I = I_{ph}$
 $= I_{sc}$ (under most conditions)

Open Circuit: $V = V_{oc}$, $I = 0$

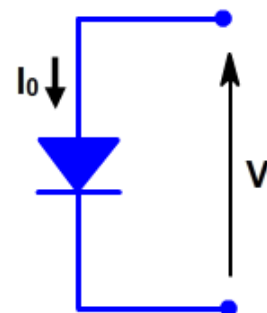
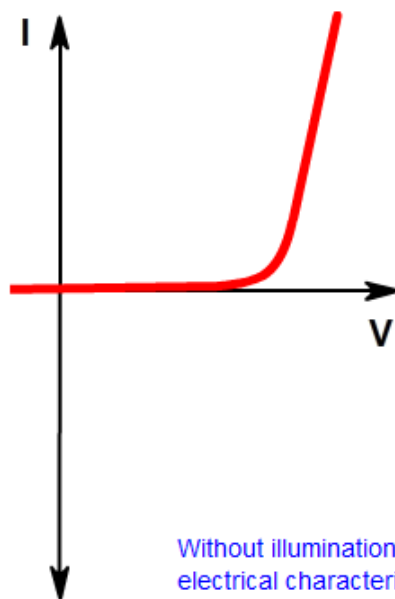
Diode Equation – Light vs Dark



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Source: adapted from Honsberg & Bowden "PVCDROM"

Diode Equation – Light vs Dark

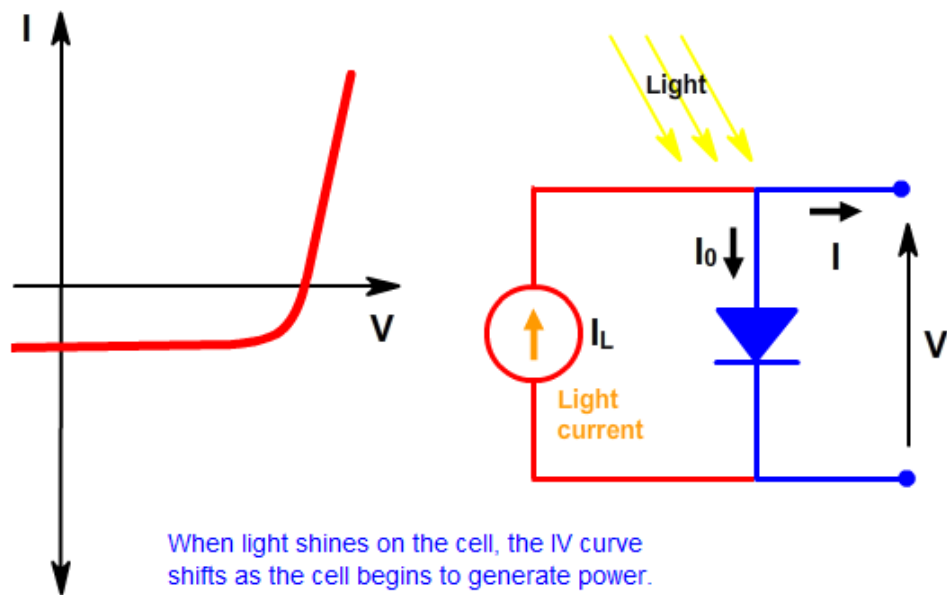


Without illumination, a solar cell has the same electrical characteristics as a large diode.

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Source: adapted from Honsberg & Bowden "PVCDROM"

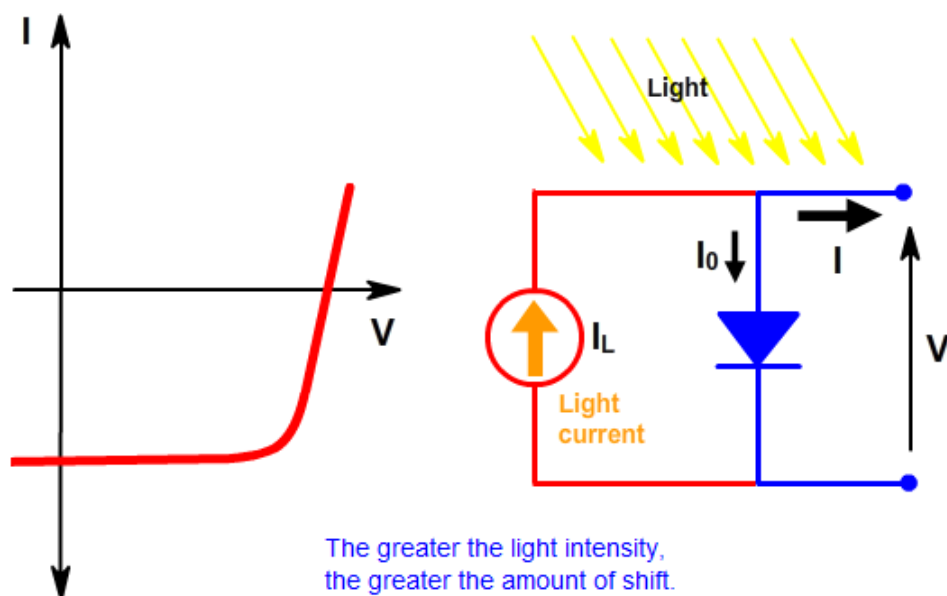
Diode Equation – Light vs Dark



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Source: adapted from Honsberg & Bowden "PVCDROM"

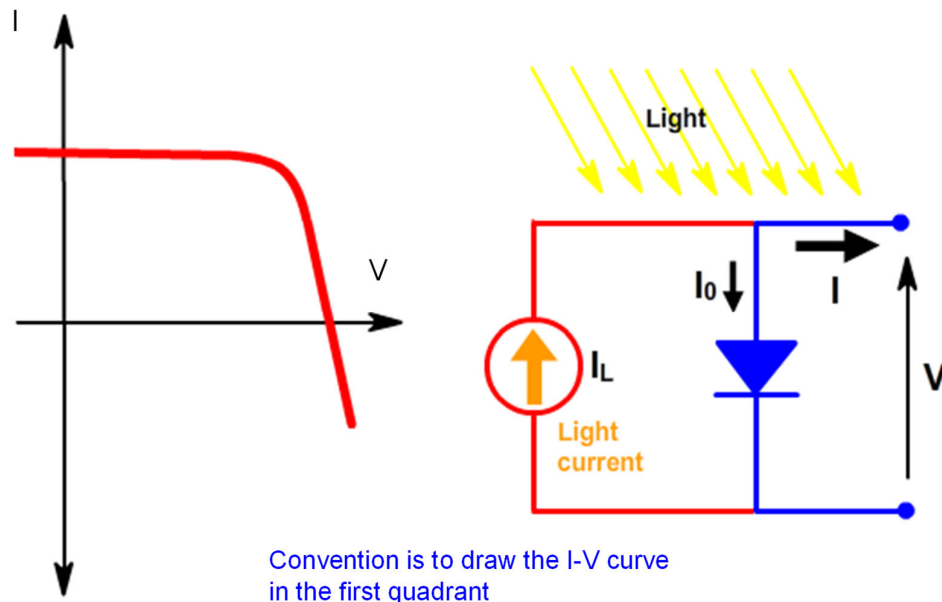
Diode Equation – Light vs Dark



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Source: adapted from Honsberg & Bowden "PVCDROM"

Diode Equation – Light vs Dark



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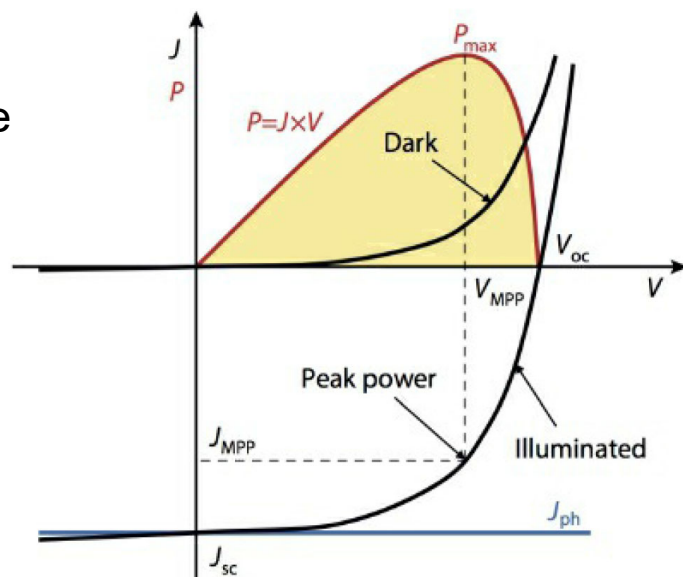
Source: adapted from Honsberg & Bowden "PVCDROM"

Photovoltaic parameters

From an illuminated I-V curve (or J-V curve)

⇒ key PV parameters:

- Short-circuit current density, J_{sc}
- Open-circuit voltage, V_{oc}
- Current density and voltage at maximum power point, J_{MPP} and V_{MPP}
- Fill factor, FF
- Peak power, P_{max}
- Conversion efficiency, η



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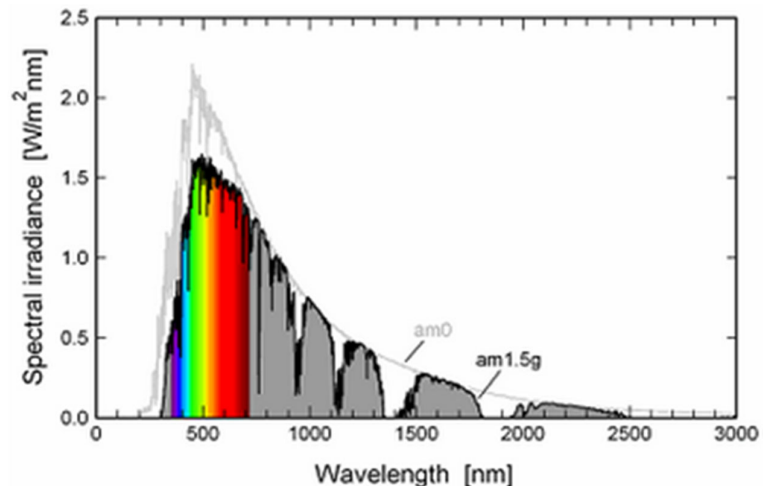
Source: textbook

Standard test conditions

- To be able to compare J-V characteristics \Rightarrow vital to perform the measurements under standard test conditions (STC)

- Three criteria for STC:

- 1) Irradiance on solar cell = 1000 W/m^2
- 2) Spectrum should be close to AM1.5G spectrum
- 3) Temperature of solar cell to be kept constant at 25°C



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Source: [https://www2.pvlighthouse.com.au/resources/courses/alternatt/The%20Solar%20Spectrum/The%20global%20standard%20spectrum%20\(AM1-5g\).aspx](https://www2.pvlighthouse.com.au/resources/courses/alternatt/The%20Solar%20Spectrum/The%20global%20standard%20spectrum%20(AM1-5g).aspx)

Short-circuit current

- $I_{sc} \equiv$ current that flows through the external circuit when the electrodes of solar cell are short circuited
- I_{sc} depends on:
 - the photon flux incident on the solar cell, determined by the spectrum of the incident light
 - the area of the solar cell is often used
 \Rightarrow in order to remove area dependence, use J_{sc} (mA/cm^2)
- Maximum J_{sc} that solar cell can deliver strongly depends on optical properties, e.g. fraction of light absorbed or reflected

Short-circuit current

- Ideal case, $J_{sc} =$ photogenerated current density, J_{ph} , with:

$$J_{ph} = qG(L_N + W + L_P)$$

where G = generation rate (assumed uniform)

L_N, L_P = minority-carriers diffusion length
for e^- and h^+ , respectively

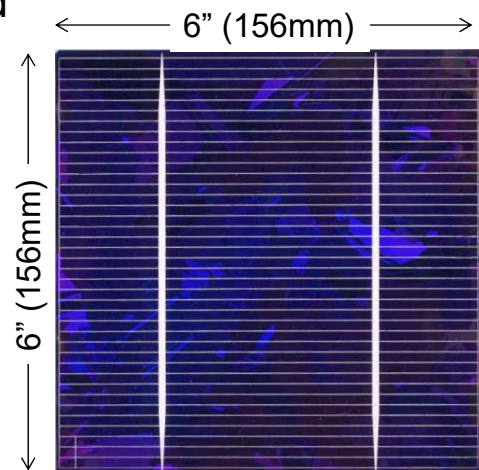
W = width of depletion region

- ⇒ Means that the only carriers that contribute to photogenerated current generated come from:
 - i) the depletion region and
 - ii) the regions up to the minority-carrier diffusion length away from the depletion region.
- ⇒ Very important consideration for solar cells design
- ⇒ Thickness of absorber layer should not be greater than the region from which the carriers contribute to J_{ph}

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Short-circuit current

- A solar cell with bandgap $E_g = 1.12\text{eV}$ – that of crystalline silicon (c-Si) – can deliver a maximum theoretical J_{sc} of 46 mA/cm^2 under AM1.5G illumination
- c-Si solar cells exhibit a J_{sc} of:
 - ~42 mA/cm^2 for lab-scale (small-area),
 - ~36 mA/cm^2 for commercially-produced large-area
- Typical commercial solar cell size:
 - $15.6\text{ cm} \times 15.6\text{ cm} = 243\text{ cm}^2$
 - ⇒ 8.5 A at 0.6 V !
 - ⇒ $P_{max} = 5\text{ W}$



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Collection Probability

Collection probability (CP) \equiv probability of carrier generated by light absorption in certain region of the device will be collected by p - n junction, i.e. contributes to photogenerated current

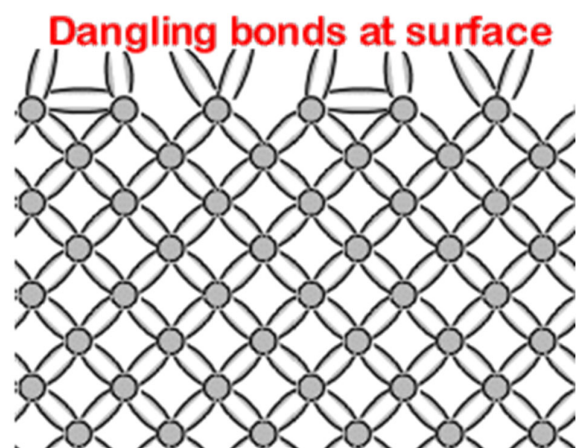
- Depends on distance that carrier must travel
 \Rightarrow within diffusion length, L ?
- CP of carriers generated in depletion region is unity as easy for e^- and h^+ to cross the junction and be collected
- Away from junction, CP drops: for carriers generated $>$ diffusion length, L , away from junction \Rightarrow CP quite low
- Similarly, if carrier is generated closer to a surface that exhibits increased recombination \Rightarrow carrier likely to recombine
 \Rightarrow Long diffusion lengths & good surface passivation important!

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Collection Probability

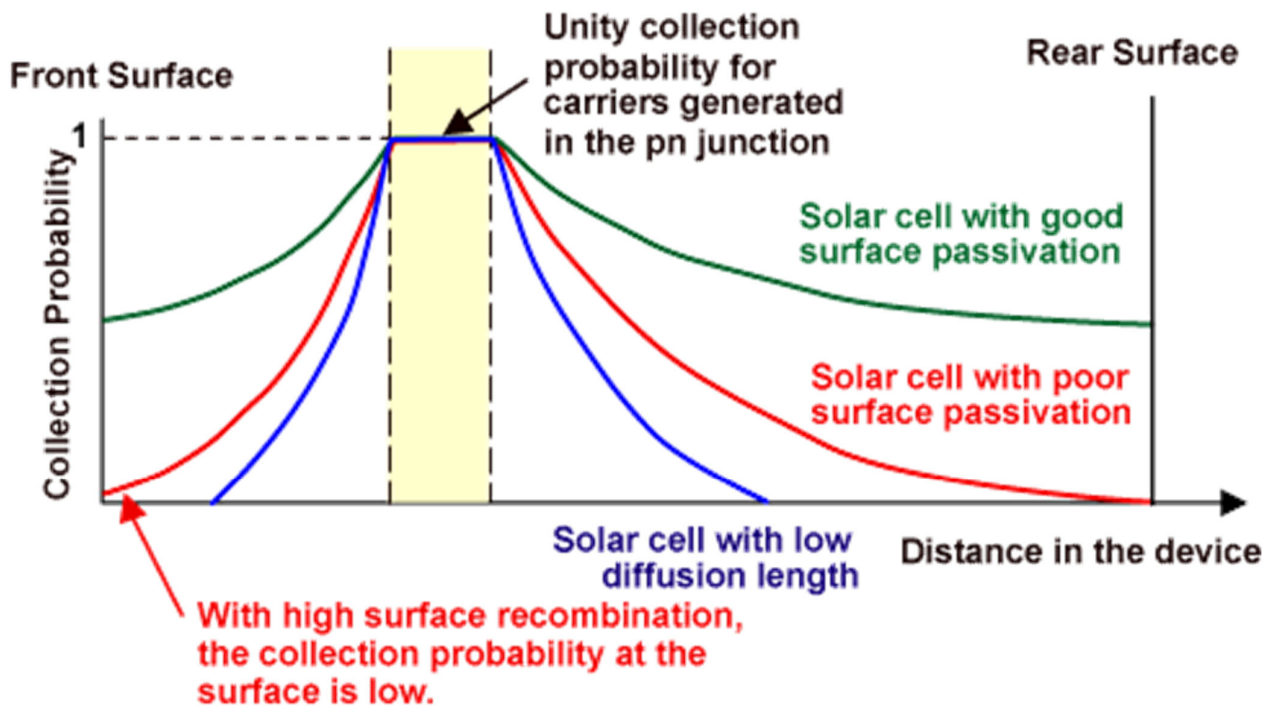
Surface passivation:

- Defected areas, such as at the surface of solar cells where the lattice is disrupted \Rightarrow recombination is very high
- Imagine: a c-Si wafer is a near-perfect single crystal, but it has been sliced, leaving a huge amount of imperfect crystal area at the top and bottom
- Interruption to periodicity of crystal lattice \Rightarrow causes defects (“dangling bonds”) at surface
- Understanding i) impacts and ii) ways to limit surface recombination \Rightarrow leads to more efficient solar cell designs



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Collection Probability



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Source: <http://www.pveducation.org/pvcdrom/solar-cell-operation/collection-probability>

Photogenerated Current Density

Integration of CP and G over device thickness W of solar cell determines photogenerated current density J_{ph} :

$$J_{ph} = q \int_0^W G(x) CP(x) dx$$

$$J_{ph} = q \int_0^W \left[\int \alpha(\lambda) \Psi_{ph} \exp(-\alpha(\lambda)x) d\lambda \right] CP(x) dx$$

where:

q = electronic charge

W = entire thickness of device

$\alpha(\lambda)$ = absorption coefficient

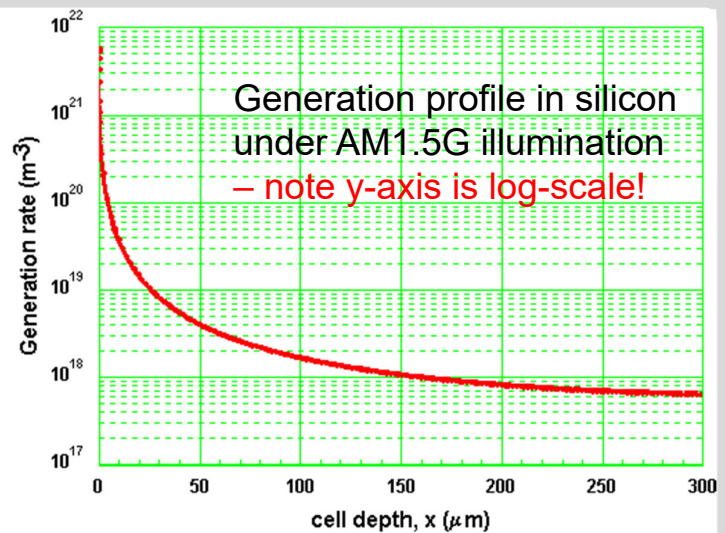
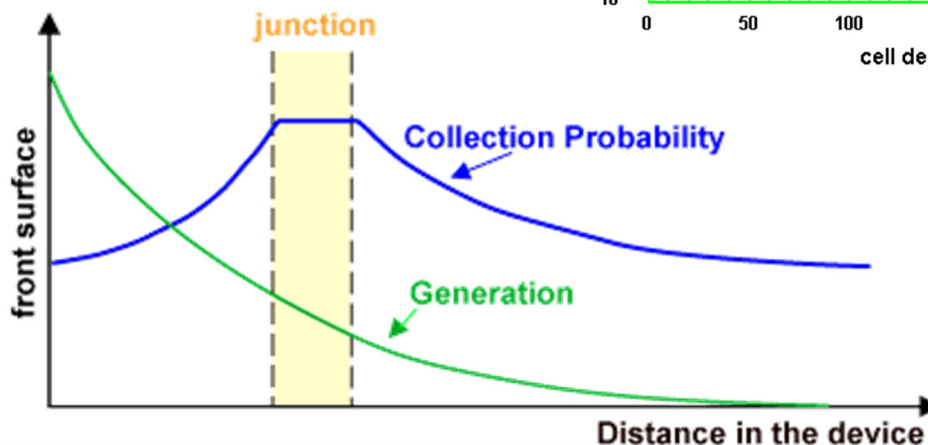
Ψ_{ph} = number of photons at each wavelength

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Photogenerated Current Density

Carrier generation rate G :
highest at front surface of
solar cell

⇒ PV devices very sensitive
to surface properties!



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Source: <http://www.pveducation.org/pvcdrom/solar-cell-operation/collection-probability>

Open-circuit voltage



- V_{oc} is the maximum voltage that the solar cell can deliver
⇒ no current flows through the external circuit
- V_{oc} corresponds to the voltage at which the dark current density compensates the photocurrent density
⇒ V_{oc} depends on J_{ph}

$$V_{oc} = \frac{k_B T}{q} \ln \left(\frac{J_{ph}}{J_0} + 1 \right) \approx \frac{k_B T}{q} \ln \left(\frac{J_{ph}}{J_0} \right)$$

where k_B = Boltzmann constant

T = temperature

J_0 = dark saturation current density

*Just reformulation
of the illuminated
diode equation*

and the approximation is justified as $J_{ph} \gg J_0$

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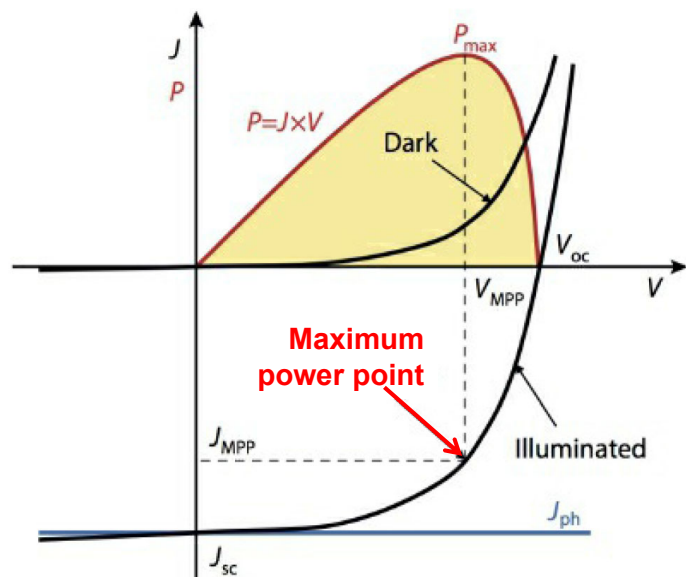
Open-circuit voltage

- The V_{oc} depends on J_{ph} and J_0 , however important to note that:
 - J_{ph} typically exhibits small variations
 - but J_0 can vary by orders of magnitude \Rightarrow depends on amount of recombination in solar cell
- Therefore, V_{oc} is an indicator of amount of recombination in the device
- Laboratory c-Si solar cells have a V_{oc} of up to 730 mV under STC
- Commercial c-Si solar cells typically have V_{oc} just over 600 mV

Fill factor

- FF \equiv ratio between the maximum power ($P_{max} = J_{mpp} \times V_{mpp}$) generated by solar cell and product of J_{sc} and V_{oc}
i.e. square with the largest area that can fit under the curve

- Maximum power point (MPP) \equiv point on J-V curve where solar cell has maximal power output
- Very important to operate PV modules at MPP
 \Rightarrow achieved via maximum power point tracking (MPPT)



Fill factor

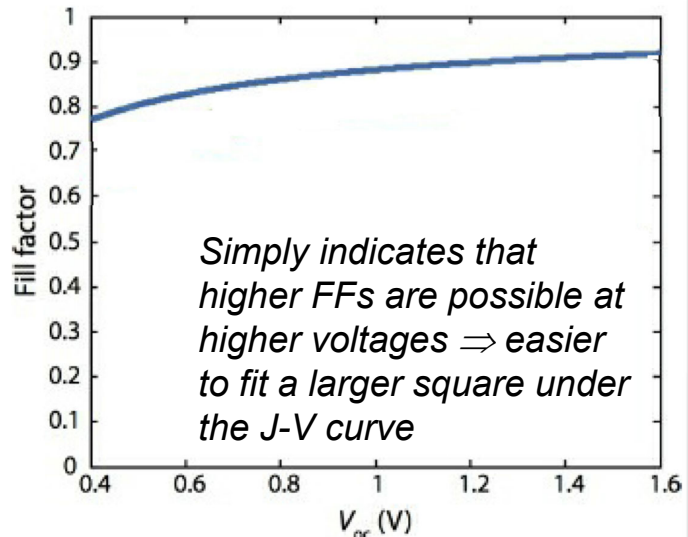
- The FF has the following *empirical* dependence on V_{oc} :

$$FF = \frac{v_{oc} - \ln(v_{oc} + 0.72)}{v_{oc} + 1}$$

where $v_{oc} = V_{oc} \frac{q}{k_B T}$

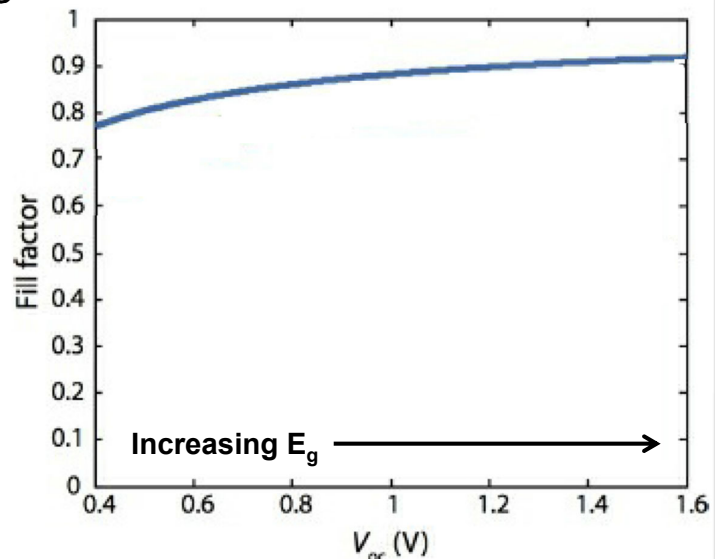
is the normalised voltage

- Good approximation for ideal value of FF when $v_{oc} > 10$
- Graph shows that FF does not change much with varying V_{oc}



Fill factor

- For solar cell with particular absorber material \Rightarrow large variations in V_{oc} are not common
- E.g. best c-Si lab device and commercial solar cell has $\Delta V_{oc} \sim 120$ mV \Rightarrow max FF of 0.85 and 0.83, respectively
- Variation in maximum FF more significant when using different absorber, e.g. GaAs ($E_g = 1.4$ eV) solar cell has FF ~ 0.89
- N.B. in practice, FF often lower due to parasitic resistive losses

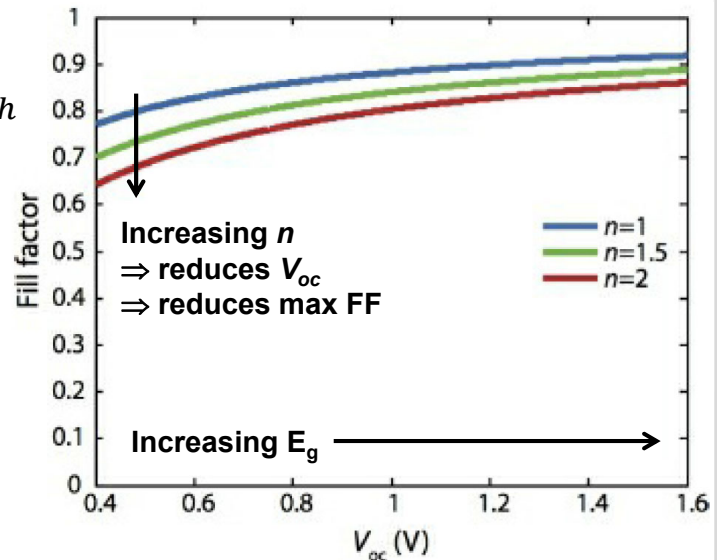


Fill factor

- In real solar cells, J_0 does not obey Boltzmann approximation
- Non-ideal diode is approximated by introducing an ideality factor n , into the illuminated diode equation:

$$J = J_0 \left[\exp \left(\frac{qV}{nk_B T} \right) - 1 \right] - J_{ph}$$

- Graph demonstrates impact of diode ideality factor on previous relationship between FF and V_{oc}



Ideality factor

- Ideality factor = measure of
 - i) junction quality and
 - ii) type of recombination in a solar cell
- Ideality factor $n = 1$ assumes that all recombination of minority carriers occurs via two mechanisms – either:
 - band-to-band recombination; or
 - recombination occurs in quasi-neutral regions (far away from junction), e.g. traps in the bulk areas of the device
- Value of ideality factor related to number of carriers the need to come together during the recombination process
- High recombination (high n value) not only lowers the FF, but also leads to low V_{oc}

Conversion efficiency

- Conversion efficiency η – or sometimes called the power conversion efficiency (PCE) – is calculated as ratio between maximal generated power (density) and incident power (density) and measured at STC ($I_{in} = 1000 \text{ W/m}^2$)

$$\eta = \frac{P_{max}}{I_{in}} = \frac{V_{mpp} \times J_{mpp}}{I_{in}} = \frac{V_{oc} \times J_{sc} \times FF}{I_{in}}$$

- Typical external parameters of commercial c-Si solar cell:

$$J_{sc} \approx 35 \text{ mA/cm}^2,$$

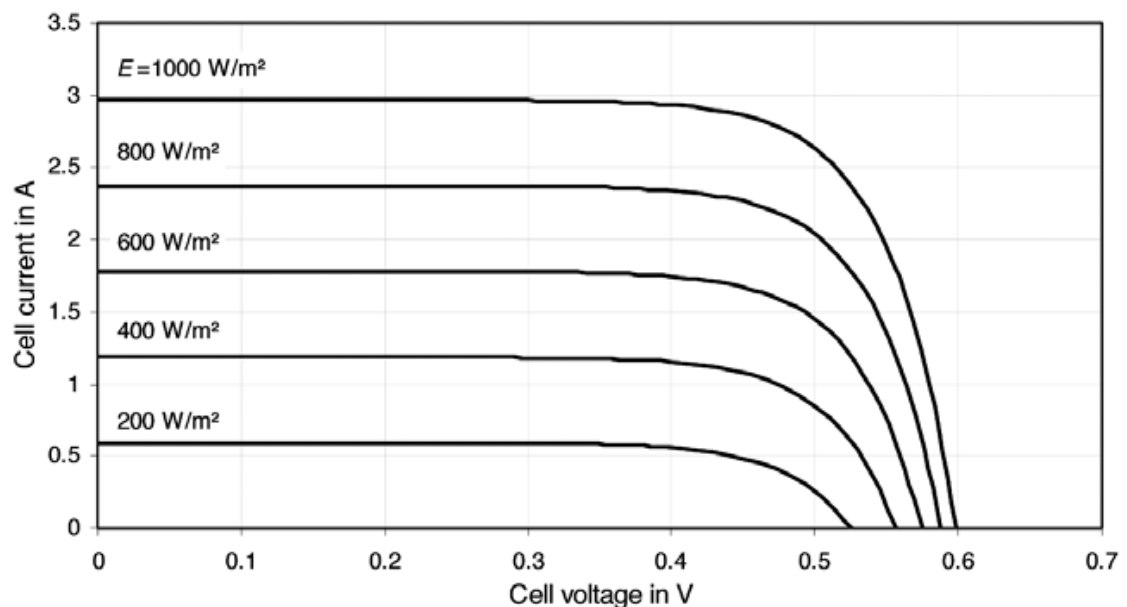
$$V_{oc} \approx 0.63 \text{ V}$$

FF in range 0.75 to 0.80

⇒ conversion efficiency is about 18%

I–V curve: effect of illumination intensity

- Photogenerated current I_{ph} scales linearly with illumination intensity



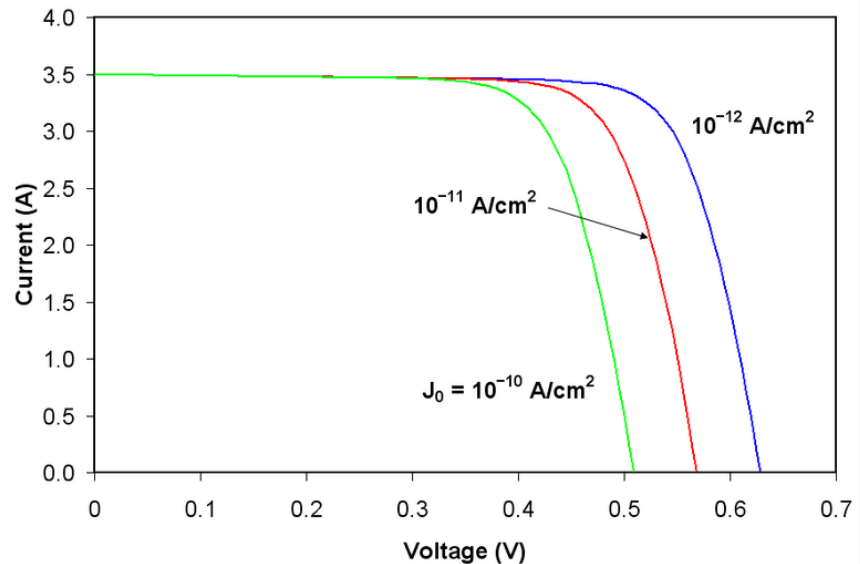
I–V curve: effect of J_0

Effect of J_0 on I–V curve \Rightarrow as J_0 decreases V_{oc} increases

- J_0 decreases as material quality increases (e.g. diffusion length)

$$V_{oc} = \frac{k_B T}{q} \ln \left(\frac{J_{ph}}{J_0} + 1 \right)$$

- J_0 increases as T increases



I–V curve: effect of temperature

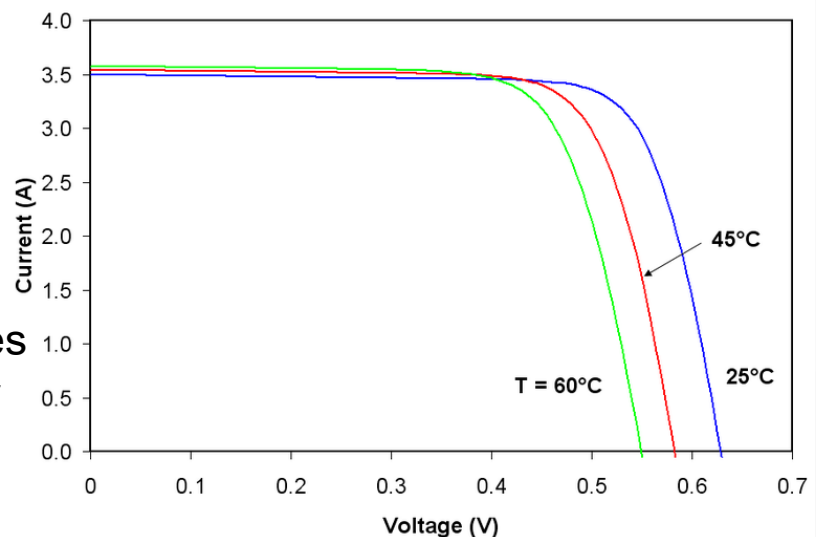
- Increasing T reduces bandgap of semiconductor and increases the energy of e^- in material

- Lower additional energy required to bridge bandgap

$\rightarrow I_{sc}$ increased slightly

\rightarrow but V_{oc} reduced greatly due to increased I_0

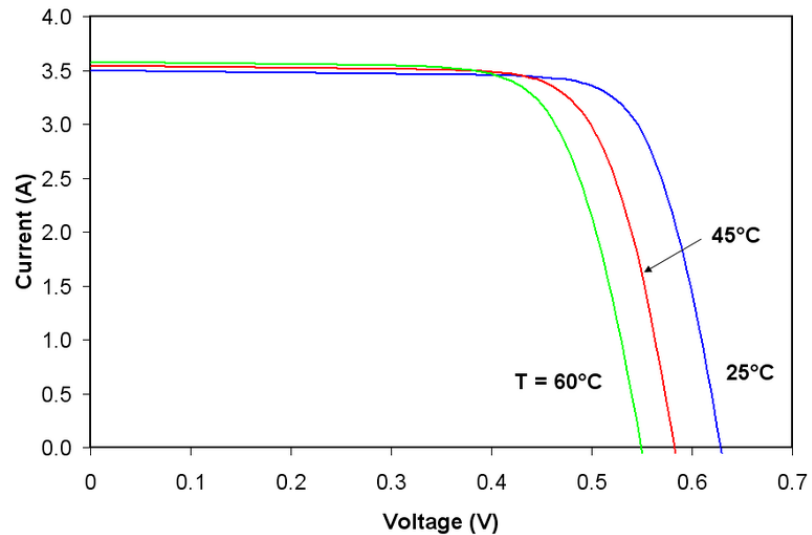
- Actual solar cell operating temperatures are often much higher than the 25°C (STC), e.g. $45 - 50^\circ\text{C}$



I–V curve: effect of temperature

For a c-Si solar cell the effect of temperature is as follows:

- V_{oc} reduces by $-2.2 \text{ mV}/^\circ\text{C}$
- I_{sc} increases very slightly with T : $+0.06\%/^\circ\text{C}$
- Reduction in FF also largely dominated by change in V_{oc} : $-0.15\%/^\circ\text{C}$
- P_{max} (η) effects are thus dominated by FF and V_{oc} : $-0.5 \text{ rel. } \%/^\circ\text{C}$

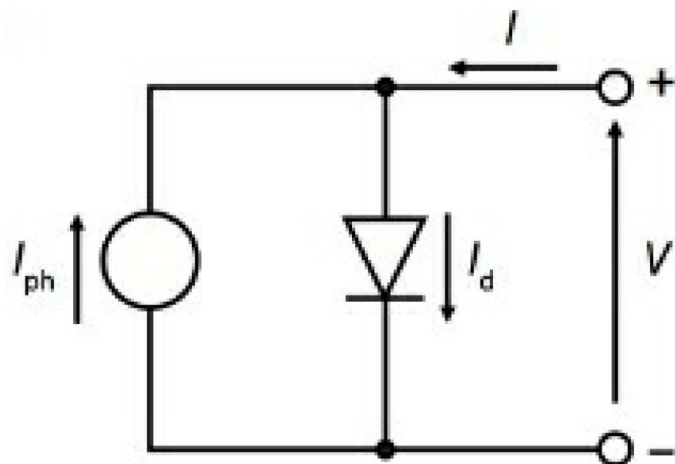


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Source: Wikipedia "Theory of Solar Cells"

Equivalent circuit

- J-V characteristic of illuminated solar cell behaves as an ideal diode ($n = 1$)
- Described by a simple equivalent circuit \Rightarrow a diode and a current source connected in parallel



$$J = J_0 \left[\exp \left(\frac{qV}{k_B T} \right) - 1 \right] - J_{ph}$$

describes the dark saturation current density of diode

describes the photogenerated current density

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Source: textbook

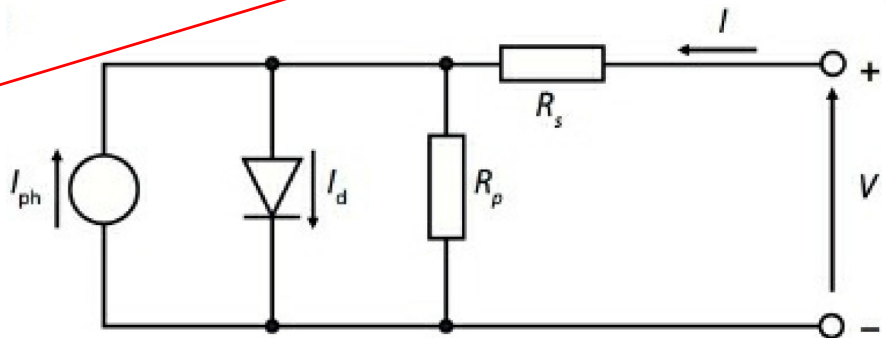
Equivalent circuit

- However, in real world two resistances exist: **series** R_s and **shunt** R_p (or R_{sh}) resistances
- The extended equivalent circuit – including both series and shunt resistances – is then given by (A = area solar cell):

$$J = J_0 \left[\exp \left(\frac{q(V - AJR_s)}{k_B T} \right) - 1 \right] + \frac{V - AJR_s}{R_p} - J_{ph}$$

Loss: voltage drop across R_s

Loss: current flow through R_p



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Source: textbook

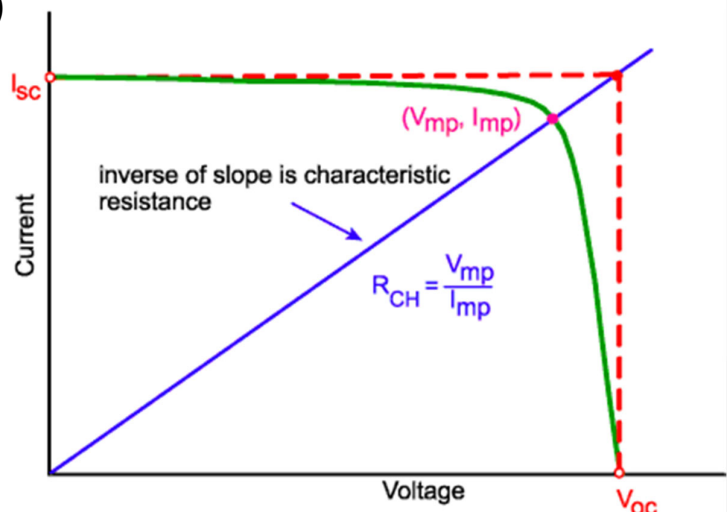
I–V curve: characteristic resistance

Characteristic resistance R_{CH} of solar cell \equiv output resistance of solar cell at MPP

If the resistance of load R_L is equal to characteristic resistance \Rightarrow maximum power is transferred to the load (solar cell operates at MPP)

$$R_{CH} = \frac{V_{mp}}{I_{mp}} \approx \frac{V_{oc}}{I_{sc}}$$

Useful parameter in solar cell analysis, esp. when examining the impact of parasitic resistances



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Source: <http://www.pveducation.org/pvcdrom/solar-cell-operation/charecteristic-resistance>

I–V curve: effect of R_s

Series resistance in a solar cell has three causes:

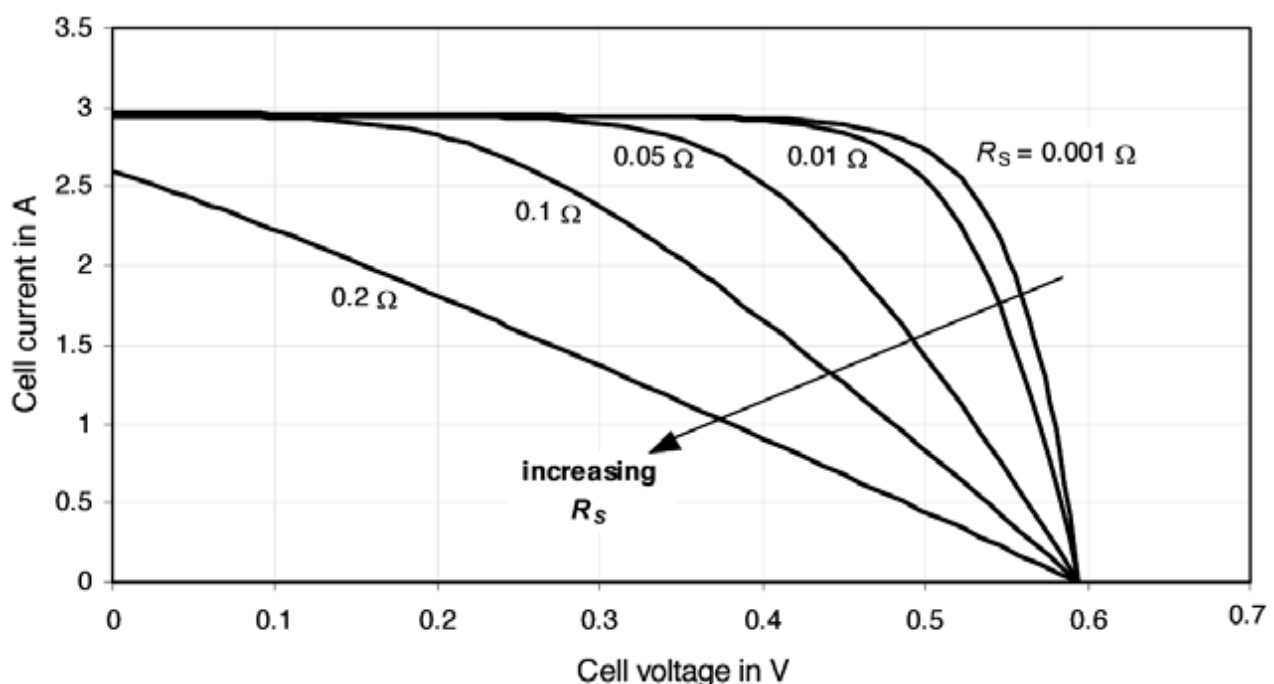
1. movement of current through emitter and base of solar cell;
2. contact resistance between silicon and metal contacts; and
3. resistance of front and rear metal contacts.

(4. and in a PV module the connectors, cables, etc.)

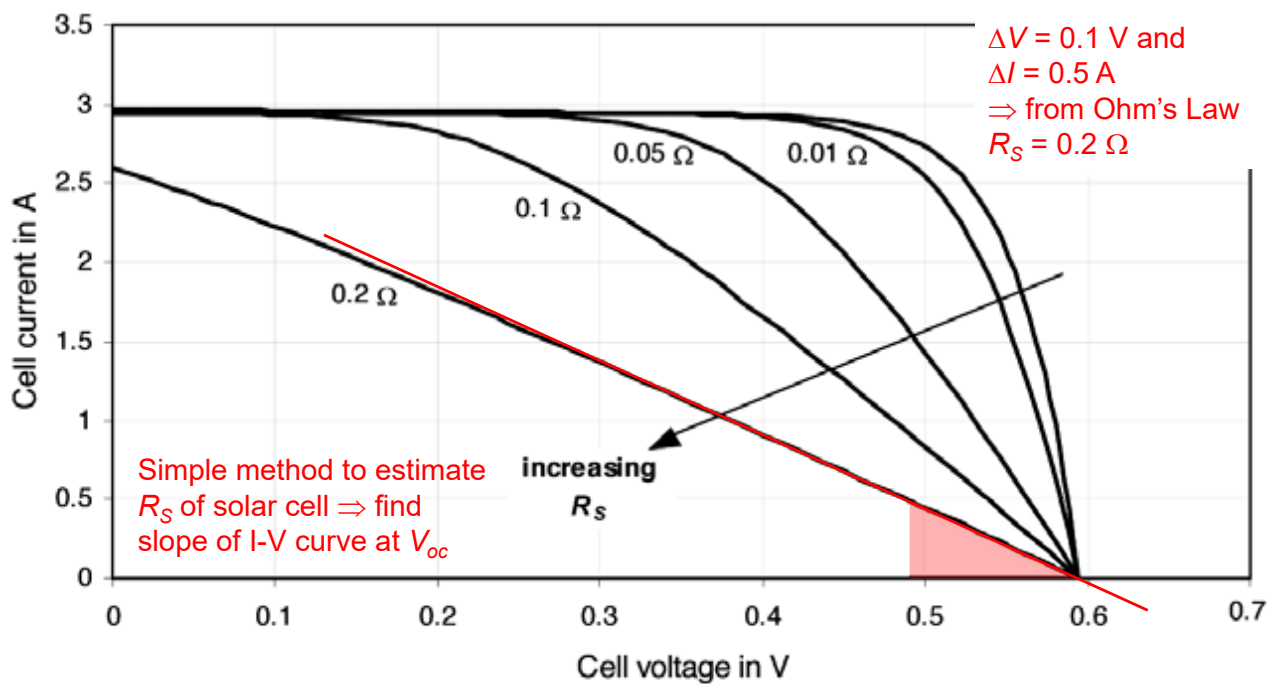
Main impact of series resistance (R_s) is to reduce the FF, although excessively high values may also reduce I_{sc}

Series resistance does not affect the solar cell at V_{oc} since the overall current flow through the solar cell (and thus through R_s) is zero, but near $V_{oc} \Rightarrow$ I-V curve can be strongly effected by R_s

I–V curve: effect of R_s



I-V curve: effect of R_s

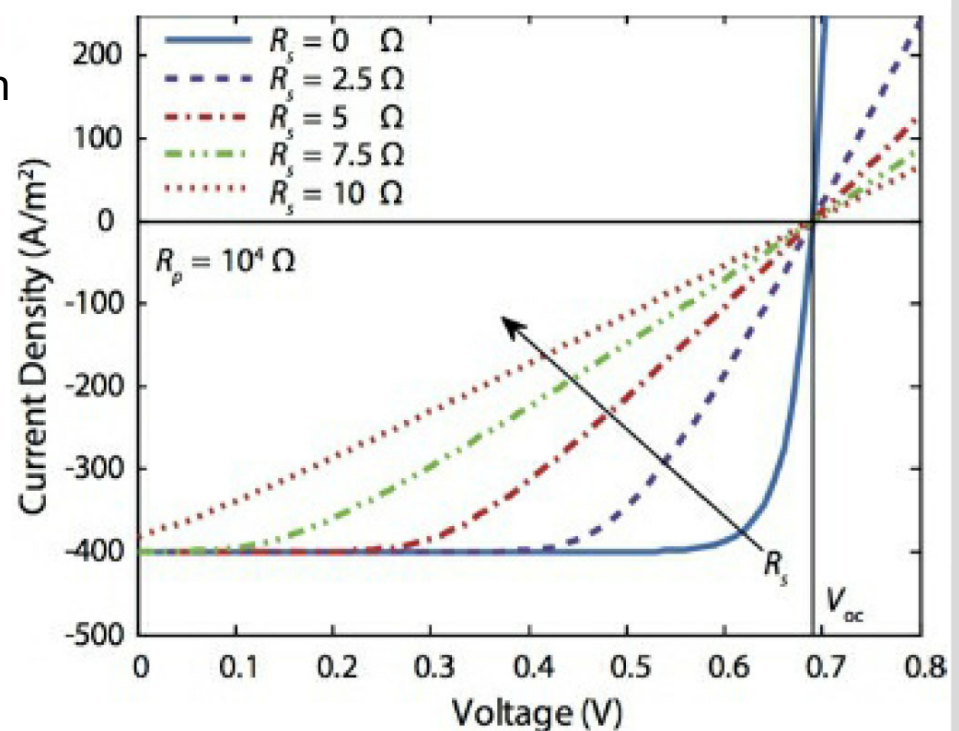


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Source: Quaschnig (2005) "Understanding Renewable Energy Systems", Earthscan

I-V curve: effect of R_s

Another example
from textbook
(note uncommon
units of A/m^2):



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Source: textbook

I–V curve: effect of R_p

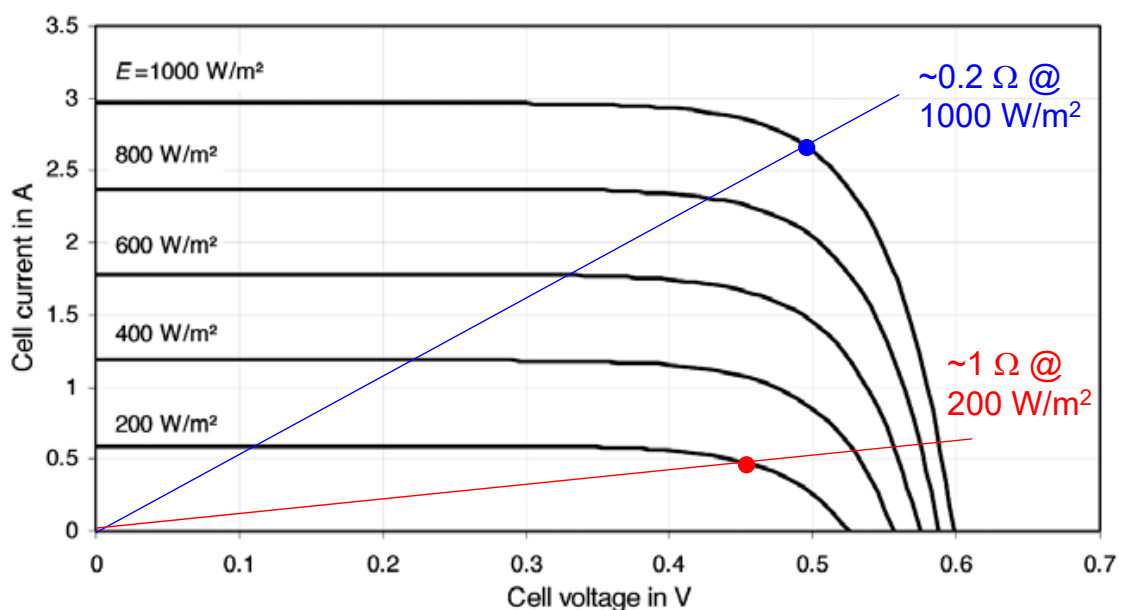
Significant power losses are caused by $R_p \Rightarrow$ but typically due to defects during solar cell fabrication (i.e. can be avoided)

Low R_p causes power loss by providing alternate current path for light-generated current \Rightarrow reduces amount of current flowing through pn -junction \Rightarrow reduces the voltage from the solar cell.

Effect of R_p is particularly severe at low light levels, since
i) there is less light-generated current and
ii) the effective resistance R_{CH} of solar cell is high
 \Rightarrow impact of R_p is larger

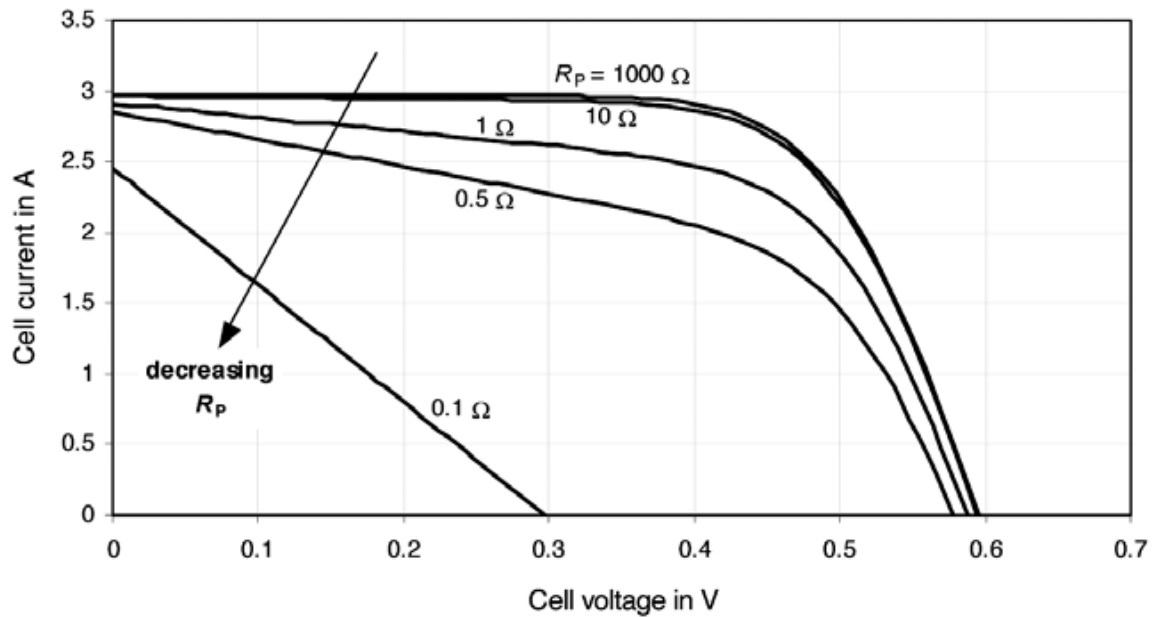
I–V curve: variation of R_{CH}

$$R_{CH} = \frac{V_{mp}}{I_{mp}} \approx \frac{V_{oc}}{I_{sc}}$$



I–V curve: effect of R_p

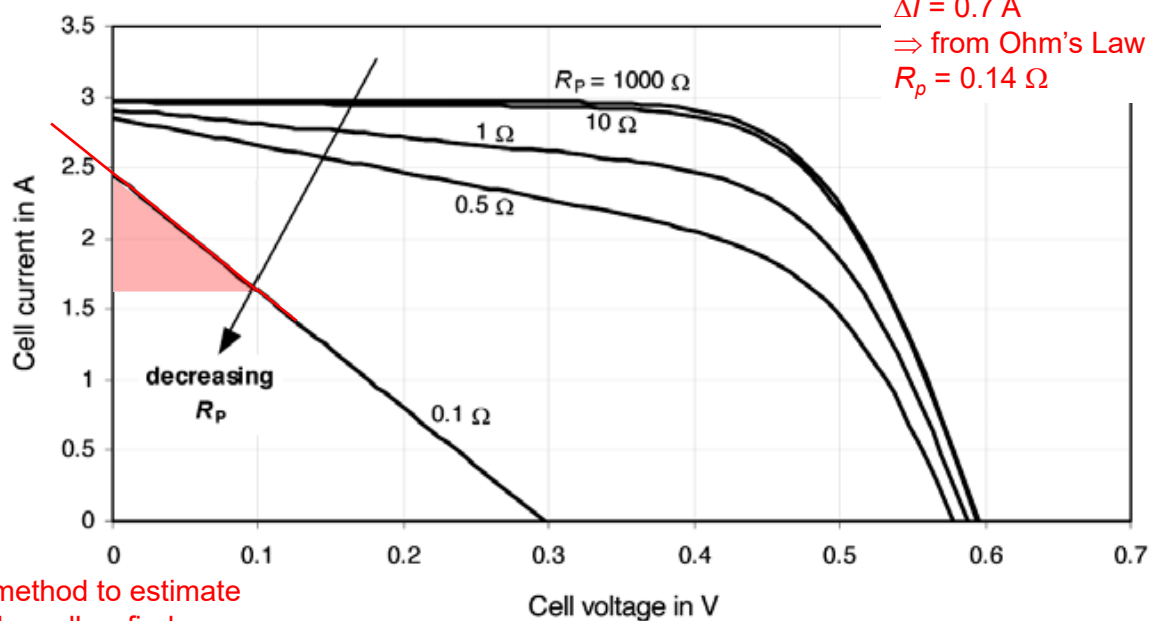
Impact of R_p first seen in FF, then in V_{oc}



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Source: Quaschnig (2005) "Understanding Renewable Energy Systems", Earthscan

I–V curve: effect of R_p



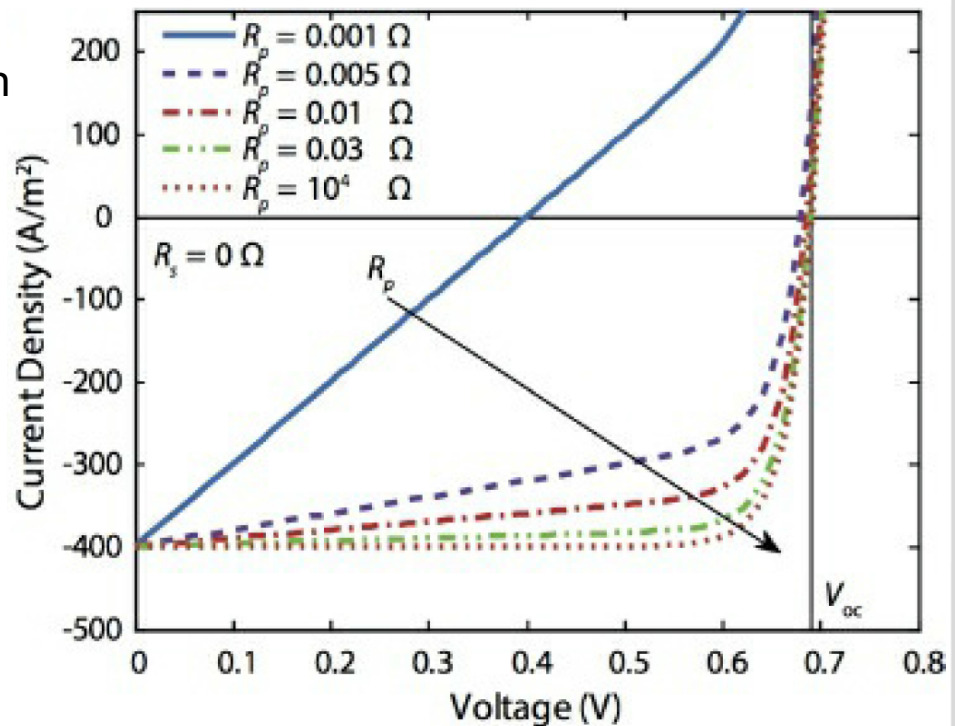
Simple method to estimate
 R_p of solar cell \Rightarrow find
slope of I-V curve at I_{sc}

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Source: Quaschnig (2005) "Understanding Renewable Energy Systems", Earthscan

I–V curve: effect of R_p

Another example
from textbook
(note uncommon
units of A/m^2):



I–V curve: effect of illumination intensity (revisited)

Changing light intensity on a solar cell changes all parameters, including I_{sc} , V_{oc} , FF , η , and impact of R_s and R_{SH}

The light intensity on a solar cell \Rightarrow called the number of “suns”, where 1 sun = standard illumination (AM1.5G, 1000 W/m^2)

E.g.

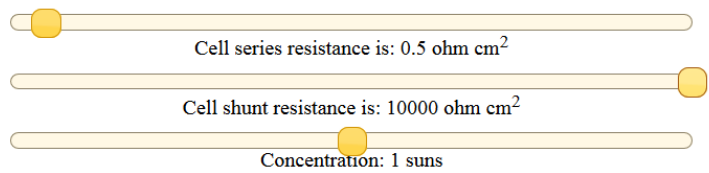
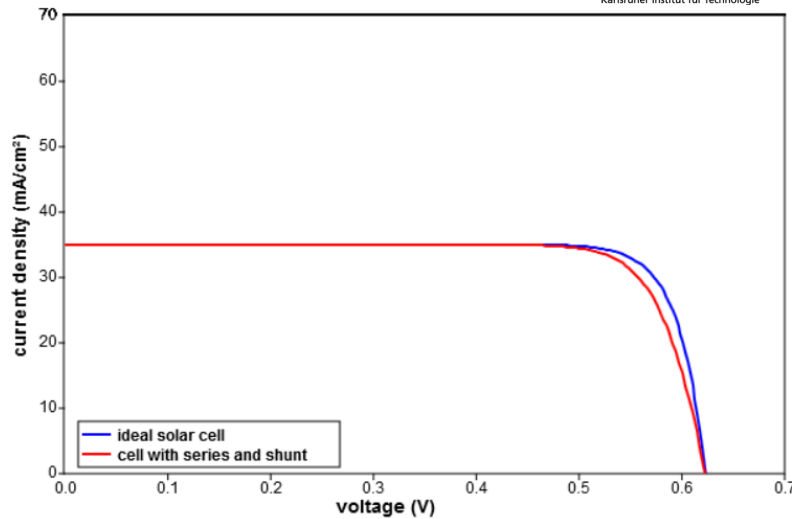
- a system with 10 kW/m^2 incident on the solar cell would be operating at 10 suns, or at 10X
- The common PV modules which are designed to operate under 1 sun conditions are called a “flat plate PV modules”, while those relying on concentrated sunlight are called “concentrator PV modules”

I–V curve: effect of illumination intensity

Good quality lab cell

- low R_S
- high R_{SH}
- 1 sun

FF = 81%



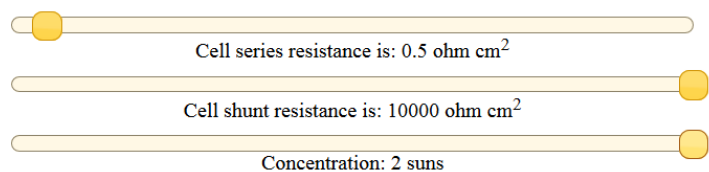
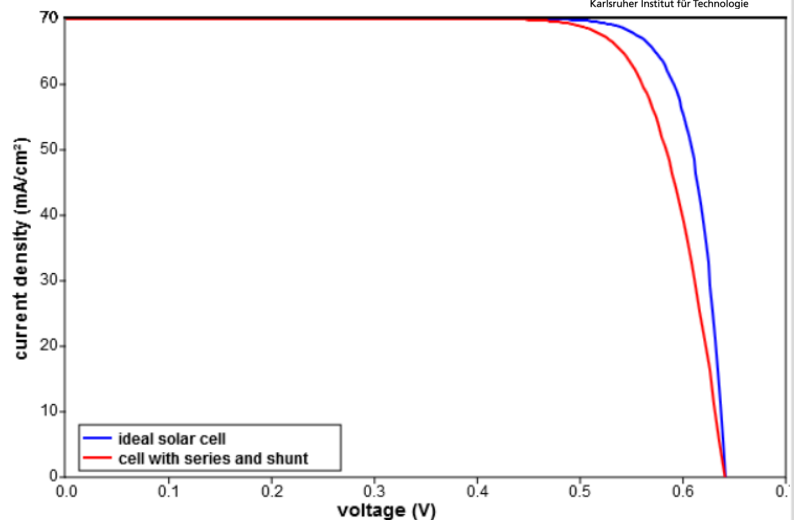
Ideal Cell: $V_{oc} = 0.623$ $I_{sc} = 35 \text{ mA/cm}^2$ FF = 0.83
Real Cell: $V_{oc} = 0.623$ $I_{sc} = 35 \text{ mA/cm}^2$ FF = 0.81

I–V curve: effect of illumination intensity

Good quality lab cell

- low R_S
- high R_{SH}
- 2 suns

FF drops slightly to 79%



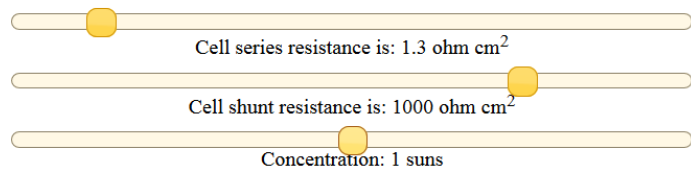
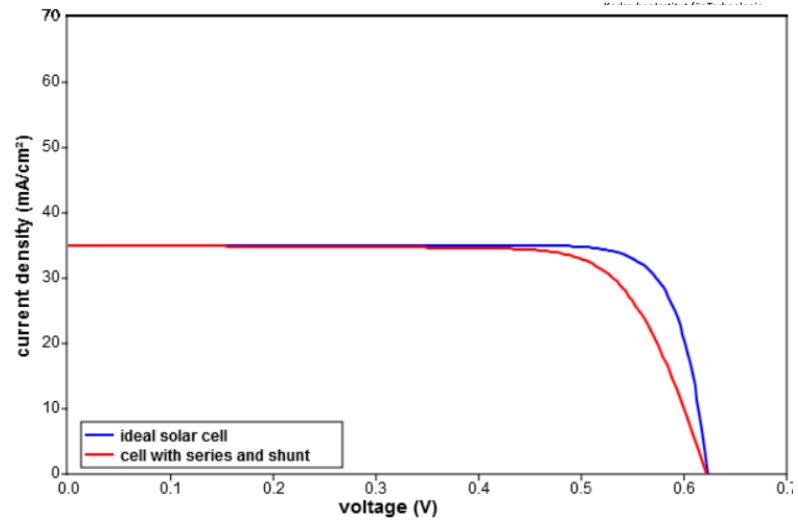
Ideal Cell: $V_{oc} = 0.641$ $I_{sc} = 70 \text{ mA/cm}^2$ FF = 0.84
Real Cell: $V_{oc} = 0.641$ $I_{sc} = 70 \text{ mA/cm}^2$ FF = 0.79

I–V curve: effect of illumination intensity

Commercial solar cell

- poorer R_S
- poorer R_{SH}
- 1 sun

FF now 76%



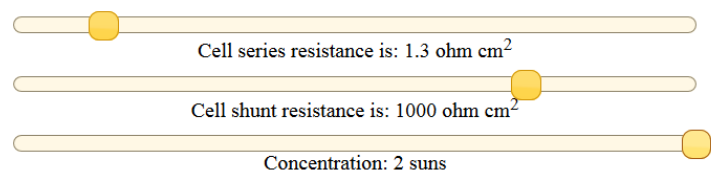
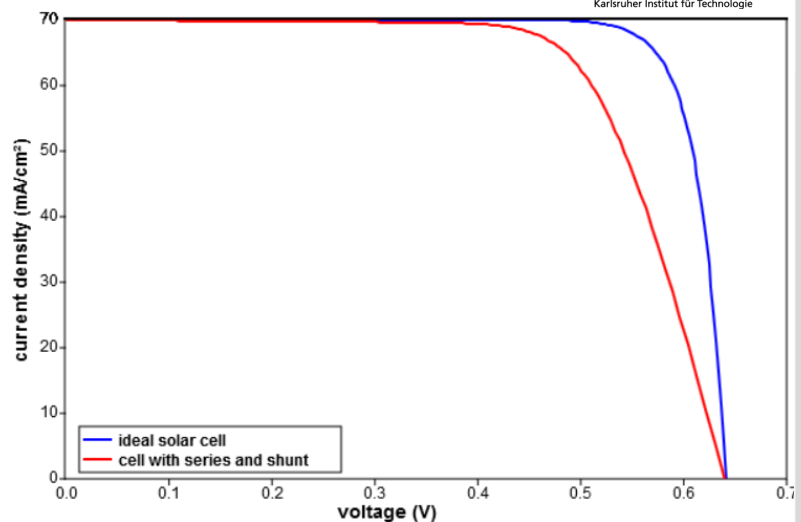
Ideal Cell: $V_{oc} = 0.623$ $I_{sc} = 35 \text{ mA/cm}^2$ $FF = 0.83$
 Real Cell: $V_{oc} = 0.622$ $I_{sc} = 35 \text{ mA/cm}^2$ $FF = 0.76$

I–V curve: effect of illumination intensity

Commercial solar cell

- poorer R_S
- poorer R_{SH}
- 2 suns

FF drops significantly to 70%



Ideal Cell: $V_{oc} = 0.641$ $I_{sc} = 70 \text{ mA/cm}^2$ $FF = 0.84$
 Real Cell: $V_{oc} = 0.64$ $I_{sc} = 69.9 \text{ mA/cm}^2$ $FF = 0.7$

I – V curve: effect of illumination intensity

Solar cell I_{sc} depends linearly on light intensity

Thus, a concentrator solar cell operating under 10 suns would have 10x times the I_{sc} as same device under 1-sun operation

⇒ but this does not the reason for increased efficiency, since the incident power also increases linearly with concentration

⇒ how can higher solar energy conversion efficiencies be achieved under concentrated sunlight then?

I – V curve: effect of illumination intensity

Answer: the efficiency benefits realised via concentrated light arise from the logarithmic dependence of the V_{oc} on I_{sc}

$$V'_{oc} = \frac{nkT}{q} \ln\left(\frac{XI_{sc}}{I_0}\right) = \frac{nkT}{q} \left[\ln\left(\frac{I_{sc}}{I_0}\right) + \ln X \right] = V_{oc} + \frac{nkT}{q} \ln X$$

So, doubling of light intensity ($X=2$) causes a 18 mV rise in V_{oc} .

I – V curve: effect of illumination intensity

Concentrators have higher efficiency potential than a 1-sun solar cell, but at what cost?

The cost of a concentrating PV system may be lower than a corresponding flat-plate PV system since a smaller area of solar cells are needed to generate same amount of power
 \Rightarrow concentrating PV is best when cost of solar cells is high and the price of the optics (lenses, mirrors) is cheap

But efficiency benefits of concentration may be reduced by increased losses in R_S as the I_{sc} increases and also the increased T operation of solar cell. Power loss due to R_S also increases as the square of light concentration
 \Rightarrow cooling systems very important!

External quantum efficiency

- $EQE(\lambda) \equiv$ fraction of photons at each wavelength that area incident on solar cell that create e^-h^+ pairs in the absorber that are successfully collected
- Measured by illuminating the solar cell with monochromatic light of wavelength λ and measuring the photocurrent I_{ph} through the solar cell:

$$EQE(\lambda) = \frac{I_{ph}(\lambda)}{q\Psi_{ph,\lambda}}$$

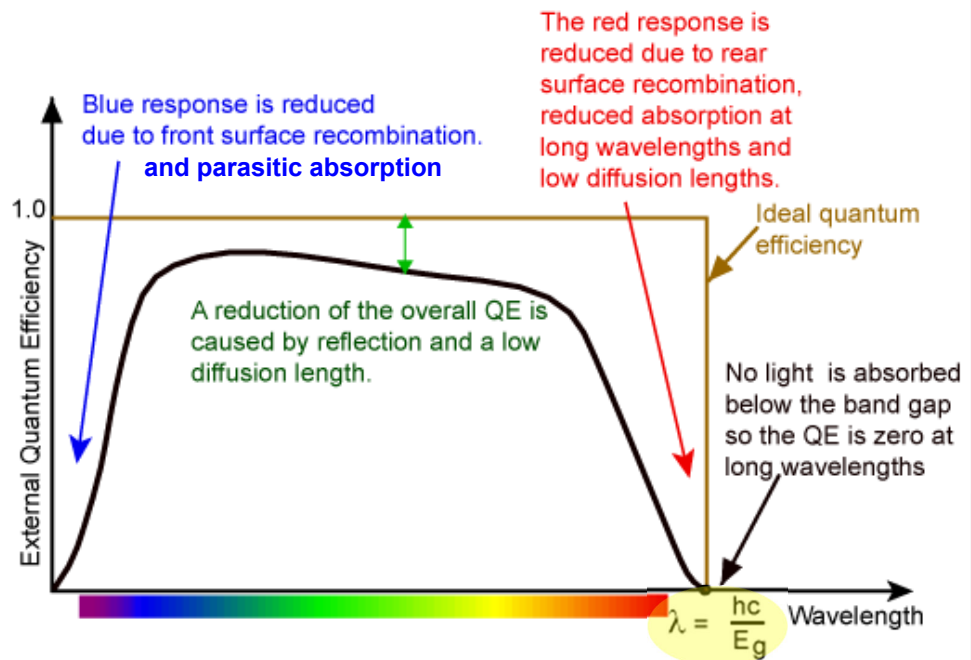
- I_{ph} determined by measuring EQE of calibrated photodiode under same light source
- Shape of EQE curve is determined by optical and electrical losses such as parasitic absorption and recombination losses

External quantum efficiency

$$\text{EQE} \equiv \frac{\text{ratio of no. carriers collected by solar cell}}{\text{no. photons incident on solar cell at given } \lambda}$$

EQE = 1 means
all photons of
certain λ are
absorbed and
resulting minority
carriers collected

EQE for photons
with energy $< E_g$
is zero



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Source: <http://www.pveducation.org/pvcdrom/solar-cell-operation/quantum-efficiency>

External quantum efficiency

EQE for practical solar cells reduced by same mechanisms that affect CP:

- blue light is absorbed very close to front surface \Rightarrow high front surface recombination will greatly reduce the QE in the blue region
- red light is absorbed in the bulk of solar cell \Rightarrow low L will reduce QE

Note that:

- EQE includes all optical losses such as transmission, parasitic absorption and reflection (ratio of $e^- - h^+$ pairs collected per incident photon)
- Often useful to consider the “internal” quantum efficiency (IQE) \equiv efficiency with which photons can generate collectable carriers after excluding optical losses (i.e. ratio of $e^- - h^+$ pairs collected per absorbed photon)
- Practically, this is achieved by measuring the reflection and transmission of the solar cell and then correcting the EQE spectrum to obtain the IQE:

$$\text{IQE} = \text{EQE} / (1 - R - T)$$

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External quantum efficiency

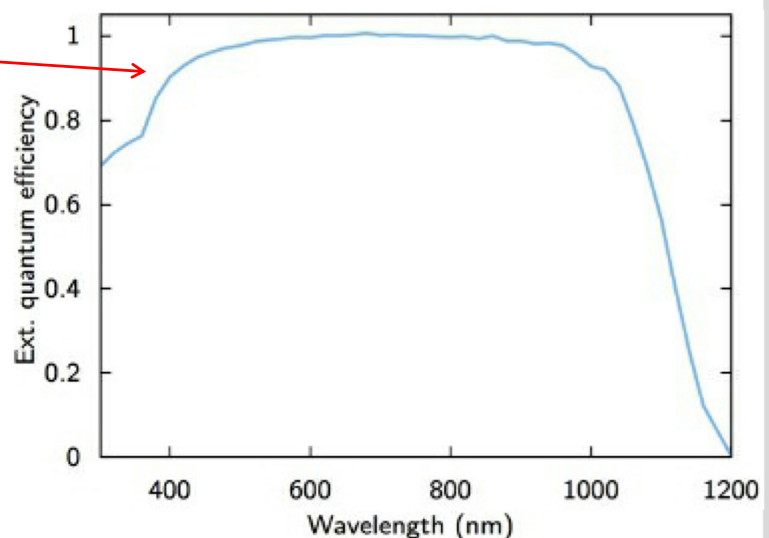
- J_{sc} can be determined by performing EQE measurement under short-circuit conditions \Rightarrow combine photon flux and EQE at each same wavelength, then integrating across all relevant wavelengths (typically 300 – 1200nm)

$$J_{sc} = -q \int_{\lambda_1}^{\lambda_2} EQE(\lambda) \Phi_{ph,\lambda} d\lambda$$

- Determining J_{sc} via EQE can have some advantages:
 - 1) it is independent of spectral shape of light source used (not like in J-V measurement)
 - 2) it can be measured in between front metal contacts

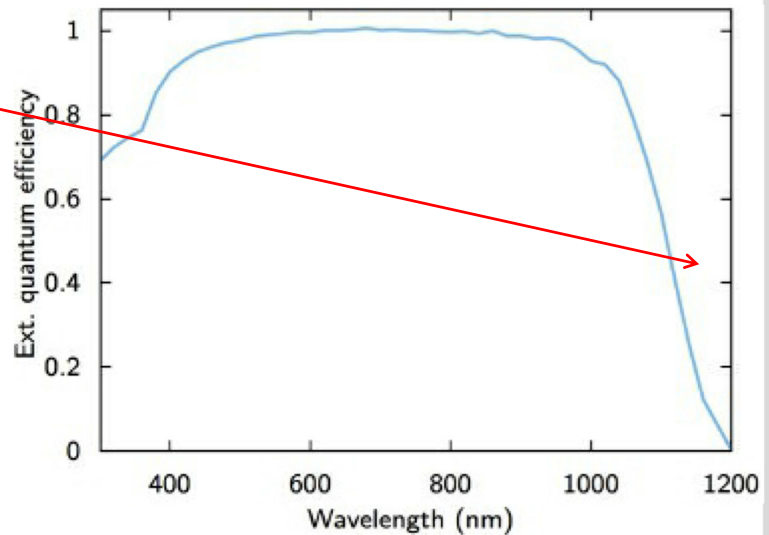
External quantum efficiency

- EQE spectrum of high-efficiency c-Si solar cell – long minority-carrier diffusion lengths and surface recombination suppressed
- Can identify major optical loss mechanisms via EQE spectrum:
- Short λ : smaller fraction of light is converted into $e^- - h^+$ pairs \Rightarrow photons absorbed in layers above the Si (parasitic absorption)



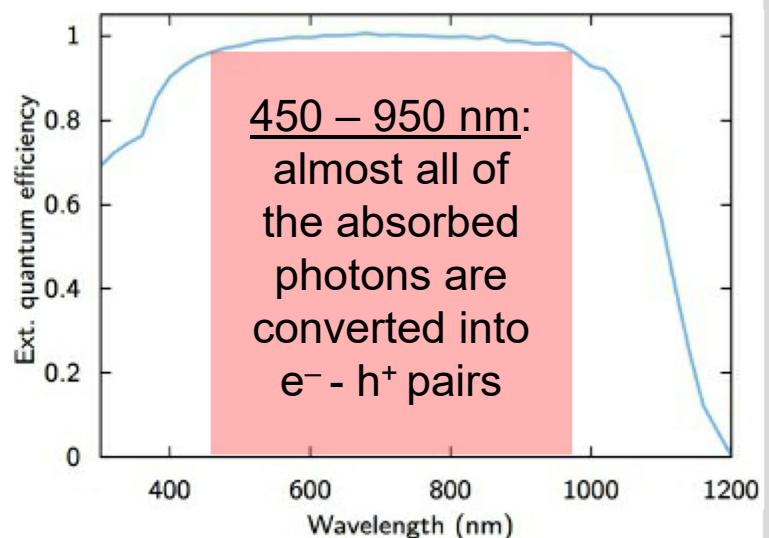
External quantum efficiency

- EQE spectrum of high-efficiency c-Si solar cell – long minority-carrier diffusion lengths and surface recombination suppressed
- Can identify major optical loss mechanisms via EQE spectrum:
- Long λ : absorber itself becomes transparent (c-Si indirect bandgap semiconductor)



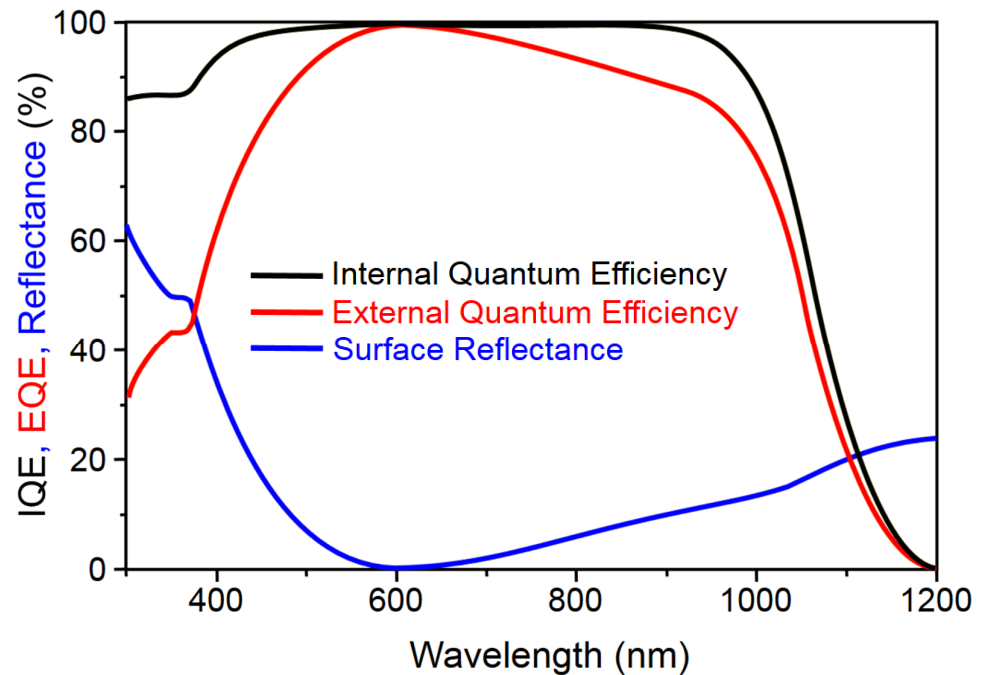
External quantum efficiency

- EQE spectrum of high-efficiency c-Si solar cell – long minority-carrier diffusion lengths and surface recombination suppressed
- Can identify major optical loss mechanisms via EQE spectrum:



External quantum efficiency

Graph showing variation of IQE, EQE, and R of crystalline silicon solar cell as function of wavelength



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Source: https://en.wikipedia.org/wiki/Quantum_efficiency

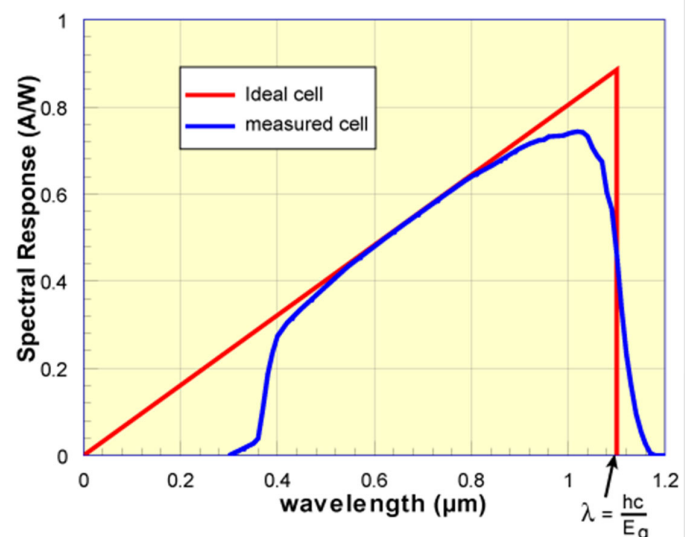
External quantum efficiency

Spectral response (SR) \equiv how much current generated by solar cell compared to power incident on it (figure-of-merit for photodetectors)

- 1) Ideal SR is limited at long λ due to E_g
- 2) SR decreases at short λ as each photon has energy $\gg E_g$
 - \Rightarrow excess energy just results in lattice thermalisation losses (heat)
 - \Rightarrow hence ratio of photons to power is reduced

- Two key loss mechanisms for single junction solar cells
- SR is related to EQE via:

$$SR = \frac{q\lambda}{hc} EQE$$



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Source: <http://www.pveducation.org/pvcdrom/characterisation/spectral-response>



Quick Test

- What is the difference between the equivalent circuit of a solar cell and the extended equivalent circuit?
- The effect of which type of resistive loss is largely avoidable during solar cell fabrication and why?
- Which solar cell parameter (V_{OC} , I_{SC} , V_{MP} , I_{MP} , FF and η) is the most affected when the following occurs:
 - Increased temperature?
 - Increased dark saturation current I_0 ?
 - Increased series resistance ?
 - Decreased shunt resistance ?
 - Decreased illumination levels (e.g. a 1/5th of a sun)?

Quick Test

- Explain how an EQE spectrum gives us information about the performance of the following regions of a solar cell (e.g. front, rear)
- Why is knowing the minority carrier diffusion length L in your semiconductor important when designing a solar cell?
- What three reasons make this location → have ideal operating conditions for a solar cell?



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Homework:

- Carefully read through chapter 9 of textbook
- Please go through worked example in Section 9.1.5
- Will help cement knowledge from last few lectures!

Example

A crystalline silicon solar cell generates a photo current density of $J_{ph} = 35 \text{ mA/cm}^2$. The wafer is doped with 10^{17} acceptor atoms per cubic centimetre and the emitter layer is formed with a uniform concentration of 10^{19} donors per cubic centimetre. The minority-carrier diffusion length in the p-type region and n-type region is $500 \times 10^{-6} \text{ m}$ and $10 \times 10^{-6} \text{ m}$, respectively. Further, the intrinsic carrier concentration in silicon at 300 K is $1.5 \times 10^{10} \text{ cm}^{-3}$, the mobility of electrons in the p-type region is $\mu_n = 1000 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$ and holes in the n-type region is $\mu_p = 100 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$. Assume that the solar cell behaves as an ideal diode. Calculate the built-in voltage, the open circuit voltage and the conversion efficiency of the cell.

$$\begin{aligned} J_{ph} &= 350 \text{ Am}^{-2} \\ N_A &= 10^{17} \text{ cm}^{-3} = 10^{23} \text{ m}^{-3} \\ N_D &= 10^{19} \text{ cm}^{-3} = 10^{25} \text{ m}^{-3} \\ L_n &= 500 \times 10^{-6} \text{ m} \\ L_p &= 10 \times 10^{-6} \text{ m} \\ D_n &= (k_B T/q) \mu_n = 0.0258 \text{ V} \times 1000 \times 10^{-4} \text{ cm}^2\text{V}^{-1}\text{s}^{-1} = 2.58 \times 10^{-3} \text{ m}^2\text{s}^{-1} \\ D_p &= (k_B T/q) \mu_p = 0.0258 \text{ V} \times 100 \times 10^{-4} \text{ cm}^2\text{V}^{-1}\text{s}^{-1} = 2.58 \times 10^{-4} \text{ m}^2\text{s}^{-1} \end{aligned}$$

Using Eq. (8.16) we calculate the built-in voltage of the cell.

$$\psi_0 = \frac{k_B T}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right) = 0.0258 \text{ V} \times \ln \left[\frac{10^{23} 10^{25}}{(1.5 \times 10^{16})^2} \right] = 0.93 \text{ V}.$$

According to the assumption that the solar cell behaves as an ideal diode, the Shockley equation describing the J - V characteristic is applicable. Using Eq. (8.25) we determine the saturation-current density.

$$\begin{aligned} J_0 &= q n_i^2 \left(\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D} \right) = 1.602 \times 10^{-19} \text{ C} (1.5 \times 10^{16})^2 \text{ m}^{-6} \\ &\quad \times \left(\frac{2.58 \times 10^{-3} \text{ m}^2\text{s}^{-1}}{500 \times 10^{-6} \text{ m} 10^{23} \text{ m}^{-3}} + \frac{2.58 \times 10^{-4} \text{ m}^2\text{s}^{-1}}{100 \times 10^{-6} \text{ m} 10^{25} \text{ m}^{-3}} \right) \\ &= 1.95 \times 10^{-9} \frac{\text{C}}{\text{m}^2\text{s}} = 1.95 \times 10^{-9} \frac{\text{A}}{\text{m}^2}. \end{aligned}$$

Using Eq. (9.1) we determine the open circuit voltage.

$$V_{oc} = \frac{k_B T}{q} \ln \left(\frac{J_{ph}}{J_0} + 1 \right) = 0.0258 \text{ V} \ln \left(\frac{350 \text{ Am}^{-2}}{1.95 \times 10^{-9} \text{ Am}^{-2}} + 1 \right) = 0.67 \text{ V}.$$

The fill factor of the cell can be calculated from Eq. (9.3). First, we normalize V_{oc} .

$$v_{oc} = V_{oc} / \frac{k_B T}{q} = \frac{0.67 \text{ V}}{0.0258 \text{ V}} = 26.8.$$

Hence,

$$FF = \frac{v_{oc} - \ln(v_{oc} + 0.72)}{v_{oc} + 1} = 0.84.$$

Finally, the conversion efficiency is determined using Eq. (9.6).

$$\eta = \frac{J_{sc} V_{oc} FF}{P_{in}} = \frac{350 \text{ Am}^{-2} \cdot 0.67 \text{ V} \cdot 0.84}{1000 \text{ Wm}^{-2}} = 0.197 = 19.7\%.$$

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Extra for Experts:

**slides from here can be
look at in your own time
and are for info only
(content will not be
examined)**

Analysis taken from Green, "Solar
Cells", Prentice Hall, 1986

***I*–*V* curve: effect of temperature**

V_{oc} decreases with T due to T -dependence of I_0 . Remembering (last lecture) the equation for I_0 from one side of a p - n junction is:

$$I_0 = qA \frac{D n_i^2}{L N_D}$$

where:

- q = electronic charge;
- D = diffusivity of the minority carrier;
- L = diffusion length of the minority carrier;
- N_D = doping; and
- n_i = intrinsic carrier concentration for silicon

Many of these parameters have *some* T -dependance, but greatest effect is due to $n_i \Rightarrow$ depends on the E_g (lower $E_g \rightarrow$ higher n_i) and on energy of carriers (higher $T \rightarrow$ higher n_i)

I–V curve: effect of temperature

Simplified equation for n_i is:

$$n_i^2 = BT^3 \exp\left(-\frac{E_{G0}}{kT}\right)$$

where:

k = Boltzmann's constant, 1.3806×10^{-23} J/K

E_{G0} = bandgap (at zero Kelvin)

B = constant (independent of temperature)

Substituting back into expression for I_0 (neglecting T -dependencies of all other parameters) gives:

$$I_0 = qA \frac{D}{LN_D} BT^3 \exp\left(-\frac{E_{G0}}{kT}\right) \approx B'T^\gamma \exp\left(-\frac{E_{G0}}{kT}\right)$$

where γ is used instead of 3 to incorporate possible T -dependencies of other materials

⇒ for Si solar cells at typical operating temperatures, I_0 approximately doubles for every 10 °C increase in temperature

I–V curve: effect of temperature

Now substituting I_0 back into equation for V_{oc} (with $E_{G0} = qV_{G0}$) and assuming that dV_{oc}/dT does not depend on dI_{sc}/dT , then (full derivation not shown here):

$$\frac{dV_{oc}}{dT} = \frac{V_{oc} - V_{G0}}{T} - \gamma \frac{k}{q}$$

⇒ demonstrates that T sensitivity of solar cell depends on V_{oc}

For Si, $E_{G0} = 1.2\text{eV}$, and using $\gamma = 3$

⇒ leads to a reduction in V_{oc} of $\sim 2.2 \text{ mV/}^\circ\text{C}$

I–V curve: effect of temperature

I_{sc} increases slightly with T since the E_G decreases and more photons can create e^-h^+ pairs. But this is a small effect and the T dependence of I_{sc} from a silicon solar cell is

$$\frac{1}{I_{sc}} \frac{dI_{sc}}{dT} \approx 0.0006 \text{ per } ^\circ\text{C for Si}$$

The T dependency of FF for Si is dominated by V_{oc} and is approximated by

$$\frac{1}{FF} \frac{dFF}{dT} \approx \left(\frac{1}{V_{oc}} \frac{dV_{oc}}{dT} - \frac{1}{T} \right) \approx -0.0015 \text{ per } ^\circ\text{C for Si}$$

The effect of T on the maximum power output P_m is

$$P_{Mvar} = \frac{1}{P_M} \frac{dP_M}{dT} = \frac{1}{V_{oc}} \frac{dV_{oc}}{dT} + \frac{1}{FF} \frac{dFF}{dT} + \frac{1}{I_{sc}} \frac{dI_{sc}}{dT}$$

$$\frac{1}{P_M} \frac{dP_M}{dT} \approx -(0.004 \text{ to } 0.005) \text{ per } ^\circ\text{C for Si}$$

I–V curve: effect of R_s

Effect on FF : for moderate values of $R_s \Rightarrow$ revised MPP approximated as power in the absence of R_s minus the power lost in R_s :

$$P'_{MP} \approx V_{MP} I_{MP} - I_{MP}^2 R_s = V_{MP} I_{MP} \left(1 - \frac{I_{MP}}{V_{MP}} R_s \right) = P_{MP} \left(1 - \frac{I_{sc}}{V_{oc}} R_s \right)$$

$$P'_{MP} = P_{MP} \left(1 - \frac{R_s}{R_{CH}} \right)$$

$$P'_{MP} = P_{MP} (1 - r_s) \quad \text{defining normalised series resistance} \quad r_s = \frac{R_s}{R_{CH}}$$

and if we assume that V_{oc} and I_{sc} are not affected by R_s then:

$$V'_{oc} I'_{sc} FF' = V_{oc} I_{sc} FF (1 - r_s)$$

$$FF' = FF (1 - r_s)$$

Typical values for area-normalized R_s range from $0.5 \Omega \cdot \text{cm}^2$ (lab cells) up to $1.3 \Omega \cdot \text{cm}^2$ (commercial solar cells)

I–V curve: effect of R_p

Maximum power approximated as power in the absence of R_{SH} minus the power lost in R_{SH}

$$P'_{MP} \approx V_{MP}I_{MP} - \frac{V_{MP}^2}{R_{SH}} = V_{MP}I_{MP} \left(1 - \frac{V_{MP}}{I_{MP}} \frac{1}{R_{SH}} \right) = P_{MP} \left(1 - \frac{V_{OC}}{I_{SC}} \frac{1}{R_{SH}} \right)$$

$$P'_{MP} = P_{MP} \left(1 - \frac{R_{CH}}{R_S} \right)$$

where we define the normalised shunt resistance as $r_{SH} = \frac{R_{SH}}{R_{CH}}$

$$V'_{OC}I'_{SC}FF' = V_{OC}I_{SC}FF \left(1 - \frac{1}{r_{SH}} \right)$$

$$FF' = FF \left(1 - \frac{1}{r_{SH}} \right)$$

Typical values for area-normalized R_S range $M\Omega \cdot \text{cm}^2$ (lab cells) down to $\sim 1000 \Omega \cdot \text{cm}^2$ (commercial solar cells)

I–V curve: effect of illumination intensity

Solar cells experience daily variations in light intensity
 \Rightarrow incident power from sun varying between 0 and 1 kW/m^2

At low light levels, the effect of R_p becomes increasingly important. Why? As light intensity decreases, the bias point and current through the solar cell also decreases, and the characteristic resistance of the solar cell increases and begins to approach R_p . When these two resistances are similar, the fraction of the total current flowing through the R_p increases, thereby increasing the fractional power loss due to R_p .

Consequently, under cloudy conditions, a solar cell with a high R_p retains a greater fraction of its original power than a solar cell with a low R_p